CONFORMALITY OR CONFINEMENT? (IR)RELEVANCE OF TOPOLOGICAL EXCITATIONS

Mithat Ünsal, SLAC, Stanford University

arXiv:0910.1245, **Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD**

arXiv:0906.5156,

arXiv:0812.2085, **Index theorem for topological excitations on R**3 x S**1**

In collaboration with E. Poppitz

Earlier collaborators on related issues:
M. Shifman, L. Yaffe

Motivations

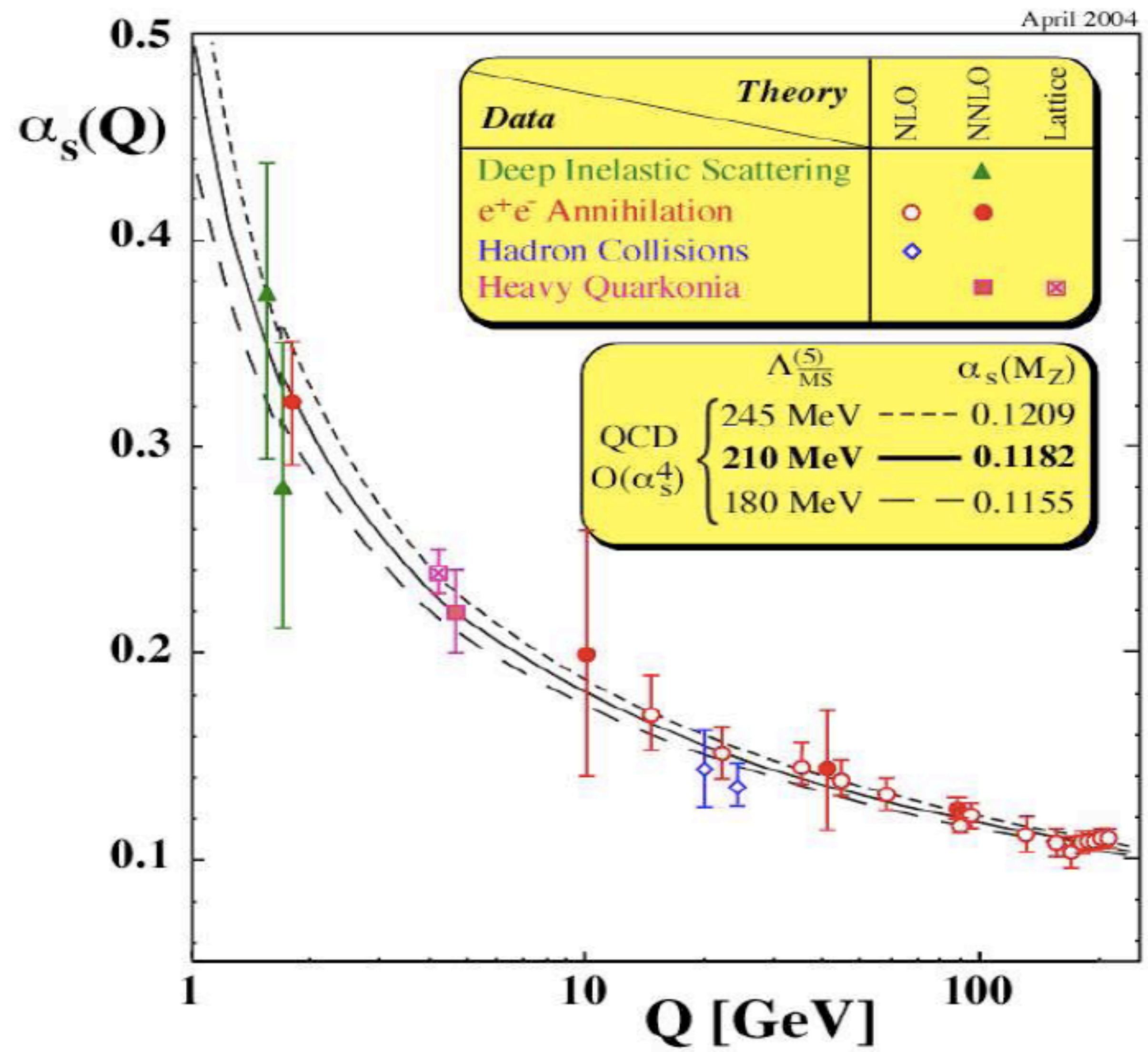
- Hadron physics, dynamics of YM and QCD-like and chiral theories
- Problem of electro-weak symmetry breaking (EWSB), scenarios in TeV scale physics. (where Higgs is a composite) to be tested at LHC.
- e.g. technicolor: naive scaled-up QCD fails with EW precision data, fails to produce acceptable spectrum. (walking, conformal...)
- Theories with interesting long distance behavior, scale invariance. (frustrated spin systems?)

GOAL

- Develop new methods to study 4d asymptotically free non-abelian gauge theories.
- Common lore: SUSY: Very friendly, beautiful. YM: Formal. QCD-like: Hard, leave it to lattice folks! (chiral limit still hard.) Non-susy chiral gauge theories: Even lattice does not work in practice.
- With adequate tools, I hope that there should not be dramatic (difficulty) differences in addressing the dynamics of YM, QCD-like, chiral or supersymmetric gauge theories.

QCD

- asymptotic freedom
- Short distance: Weakly coupled, calculable...
- Long distance ??? (Lattice works, analytical methods gloomy)

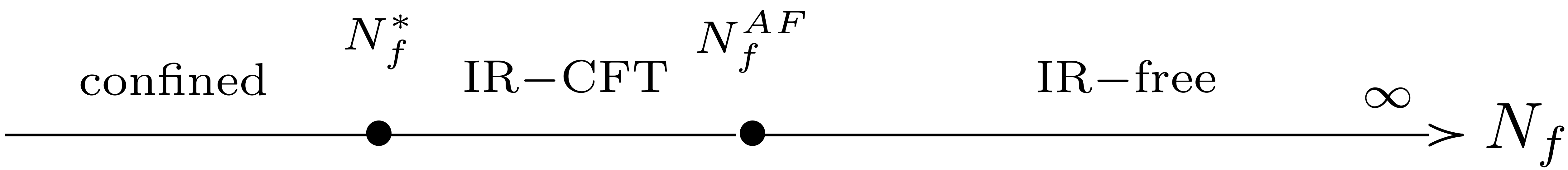


from: S. Bethke, hep-ex/0407021

- There are also asymptotically free gauge theories which exhibit different long distance behavior, such as absence of confinement. What is the fundamental difference?

Conformality or confinement:

Phases of non-abelian gauge theories



Conceptually, two problems of out-standing importance in gauge theories:

Mechanisms of confinement and conformality:
What distinguishes two theories, one just below the conformal boundary and confines, and the other slightly above the conformal window boundary? In other words, why does a confining gauge theory confine and why does an IR-CFT, with an almost identical microscopic matter content, flows to a CFT?

Lower boundary of conformal window: What is the physics determining the boundary of conformal window?

Outline

- Progress in understanding of confinement mechanism(s) in vector-like and chiral theories
- Generalized QCD (different physical IR behavior, scale invariance)
- Conformality or confinement? (and what determines it?)
- Other applications (chiral gauge theories, susy)

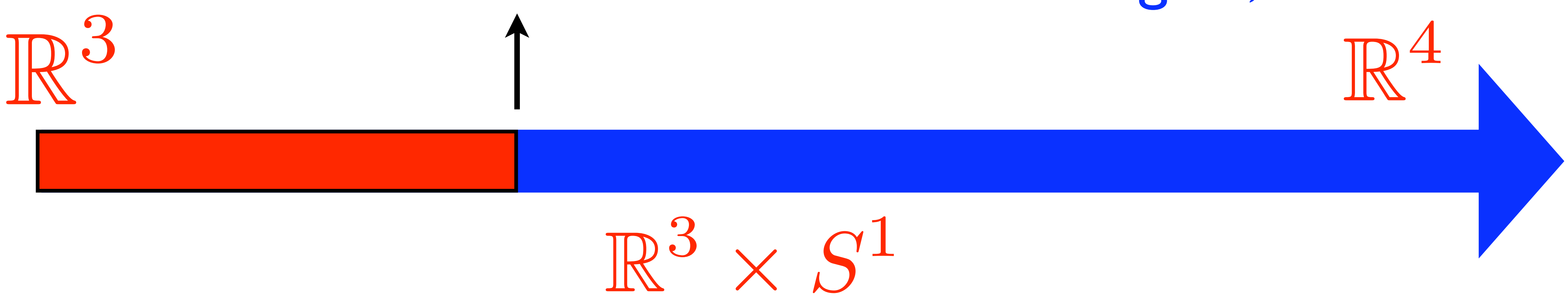
A broad-brush overview of some recent progress.

$\mathbb{R}^4 \Longrightarrow \mathbb{R}^3 \times S^1$ Locally 4d.

Take advantage of circle (as control parameter) AF.

Traditional: thermal compactification

Phase transition: Bad for our goal, but inevitable.



However: There are useful ways to go around this thermal impass.

“NEW” METHODS

1) Twisted Partition Function
Circle compactification--pbc for fermions

2) Deformation theory

a small step in the desired direction: One of the two always guarantees that small and large circle physics are connected in the sense of center symmetry and confinement.

CIRCLE COMPACTIFICATION

why bother?

various "deformations" of 4d field theories have been useful to study aspects of nonperturbative dynamics.

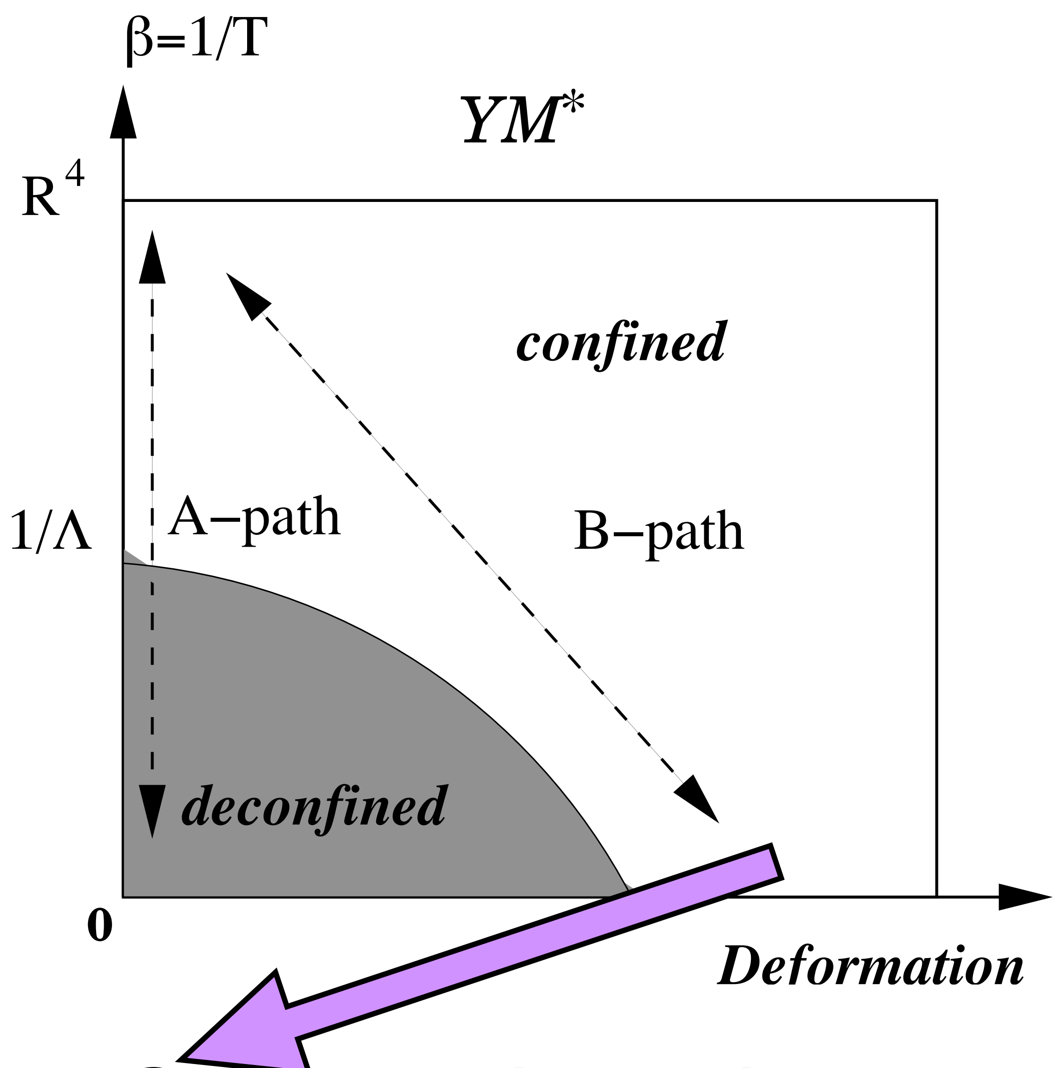
especially true in supersymmetry, where consistency with all calculable deformations play an important role, e.g.:

- circle compactification of N=2 4d SYM
(Seiberg, Witten, 96)
- circle compactification of N=1 4d SYM
(Aharony, Intriligator, Hanany, Seiberg, Strassler, 97;
Dorey, Hollowood, Khoze, Mattis, 99)

in the supersymmetric case, using holomorphy, one argues that with supersymmetric b.c. there is a smooth 4d limit

for nonsupersymmetric theories, its utility is understood recently.

Raison d'être of deformation theory at finite N

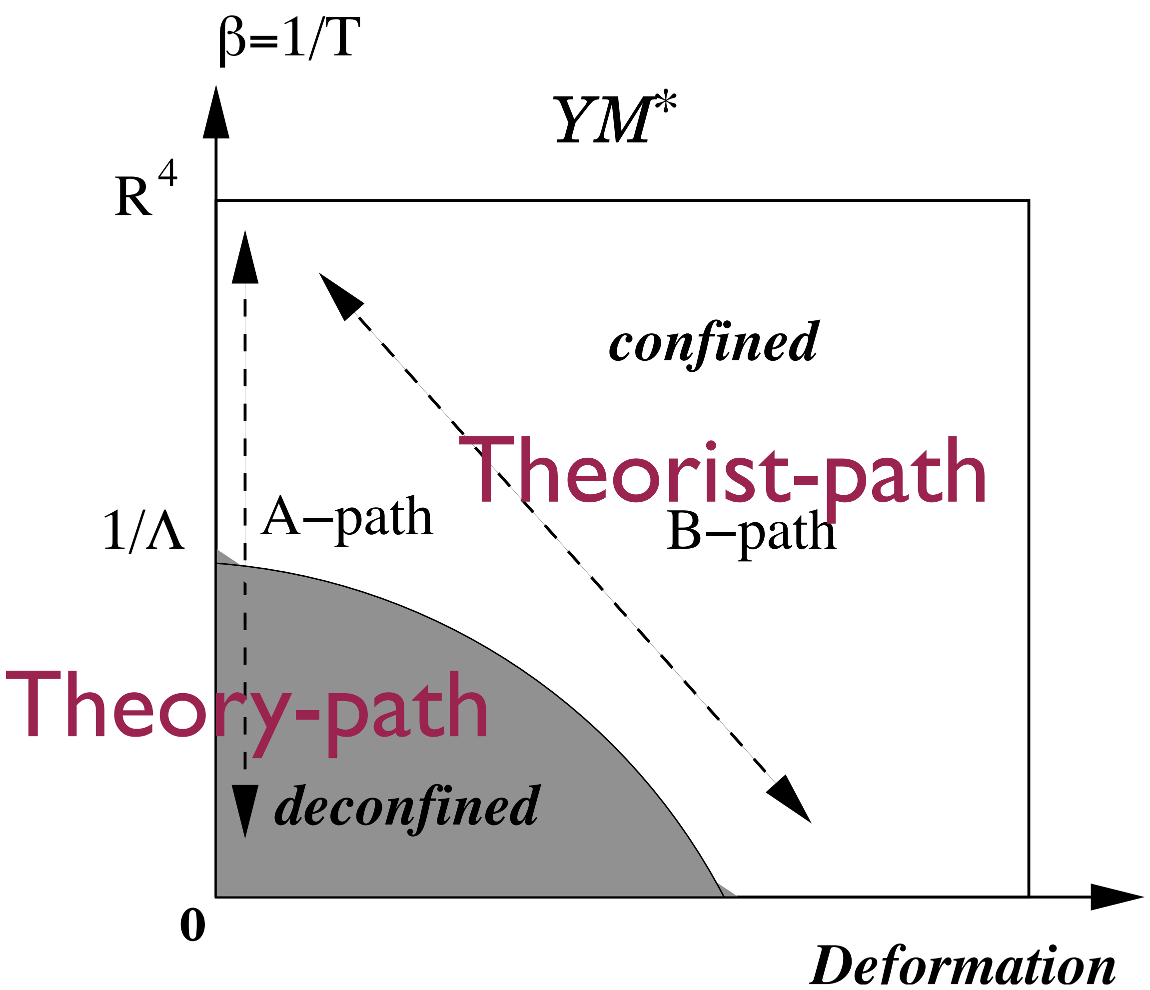


Smooth connection to the target theory. A new method to avoid singularities.

One can show the mass gap and linear confinement (similar to Seiberg-Witten and Polyakov solutions). Although the region of validity does not extend to large circle, it is continuously connected to it with no gauge invariant order parameter distinguishing the two regimes.

Deformed YM theory at finite N

Shifman-MU, Yaffe-MU



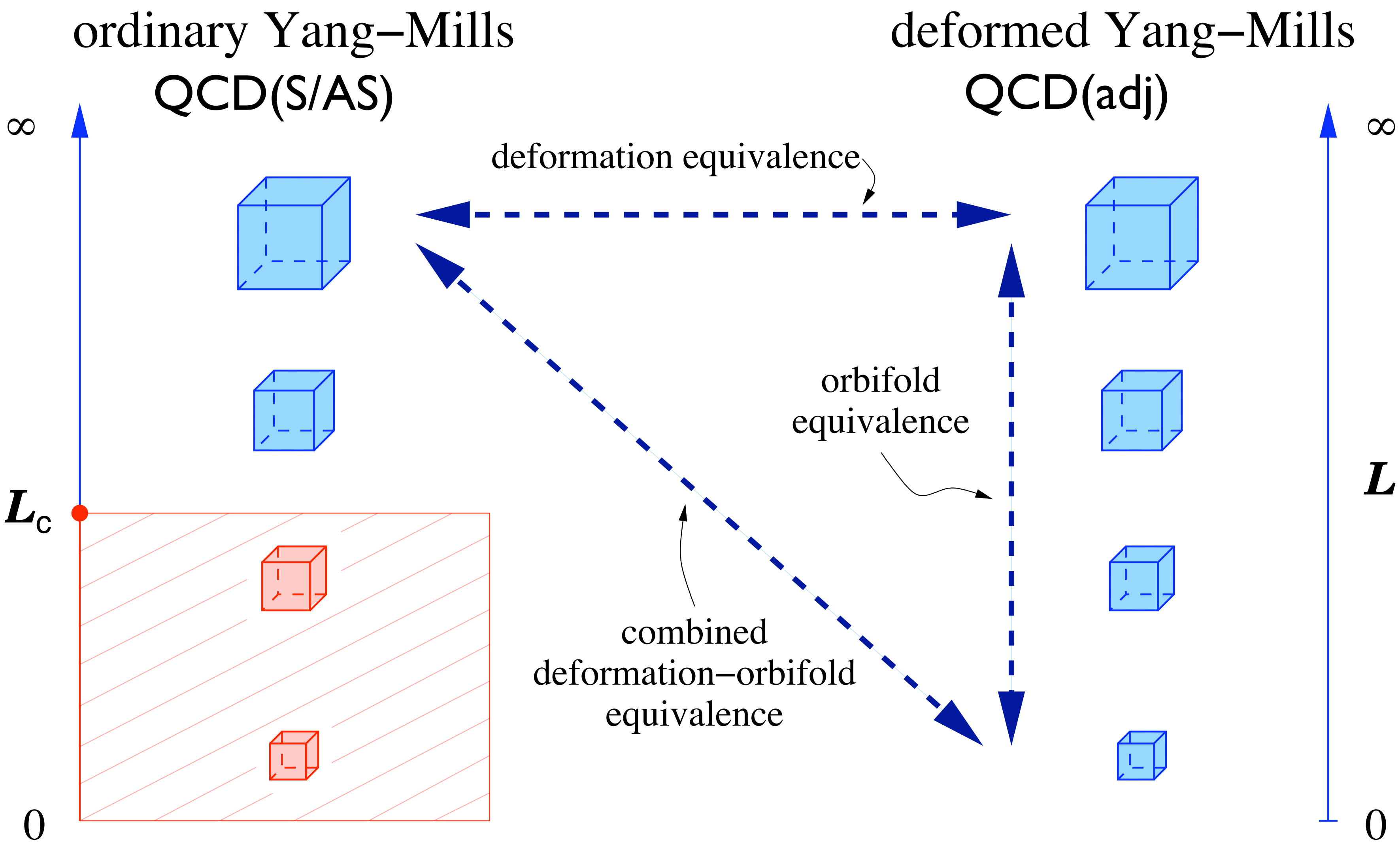
$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left| \mathrm{tr}\,(U^n) \right|^2$$

Lattice studies by Ogilvie, Myers, Meisinger backs-up the smoothness conjecture.

Ogilvie, Myers also independently proposed the above deformations to study phases of partially broken center.

Raison d'être of deformation theory at infinite N



80's: EK, QEK, TEK...
Eguchi-Kawai, EK,
Gonzalez-Arroyo, Okawa, TEK,
Bhanot, Heller, Neuberger, QEK,
Gross-Kitazawa
Parisi et.al.
Tan et.al

Last few years:
QCD(adj), deformations:

Pavel Kovtun, MU, Yaffe,
Shifman, MU.
Barak Bringoltz, Sharpe,
D'elia et.al,
Bedaque et.al.

Homage to source of inspiration:
At large N, volume independence is an exact property, a theorem.
Solution of small volume theory implies the solution of the theory on R4.
First working example of EK-reduction (25 years after the birth of idea)
QCD(Adj) with pbc: The most insightful/friendly QCD-like theory.

SU(N) QCD(adj)

$$S = \int_{R^3 \times S^1} \frac{1}{g^2} \mathrm{tr} \left[\frac{1}{4} F_{MN}^2 + i \bar{\lambda}^I \bar{\sigma}^M D_M \lambda_I \right]$$ short distance

Center Z_{N_c}

Chiral $(SU(n_f) \times Z_{2N_c n_f})/Z_{n_f}$

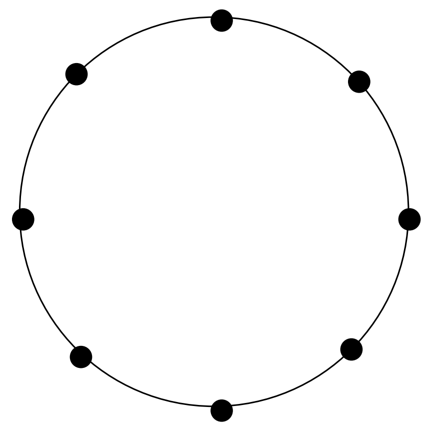
Solve it by using twisted partition function.

techicolor: minimal walking for 4 flavors?

AF-boundary: 5.5 flavors

Spatial Wilson line/non-thermal Polyakov loop

With deformation or pbc for adjoint fermions, eigenvalues repel. Minimum at



$$U = \mathrm{Diag}(1, e^{i2\pi/N}, \ldots, e^{i2\pi(N-1)/N})$$

$$\langle \mathrm{tr} U \rangle = 0$$

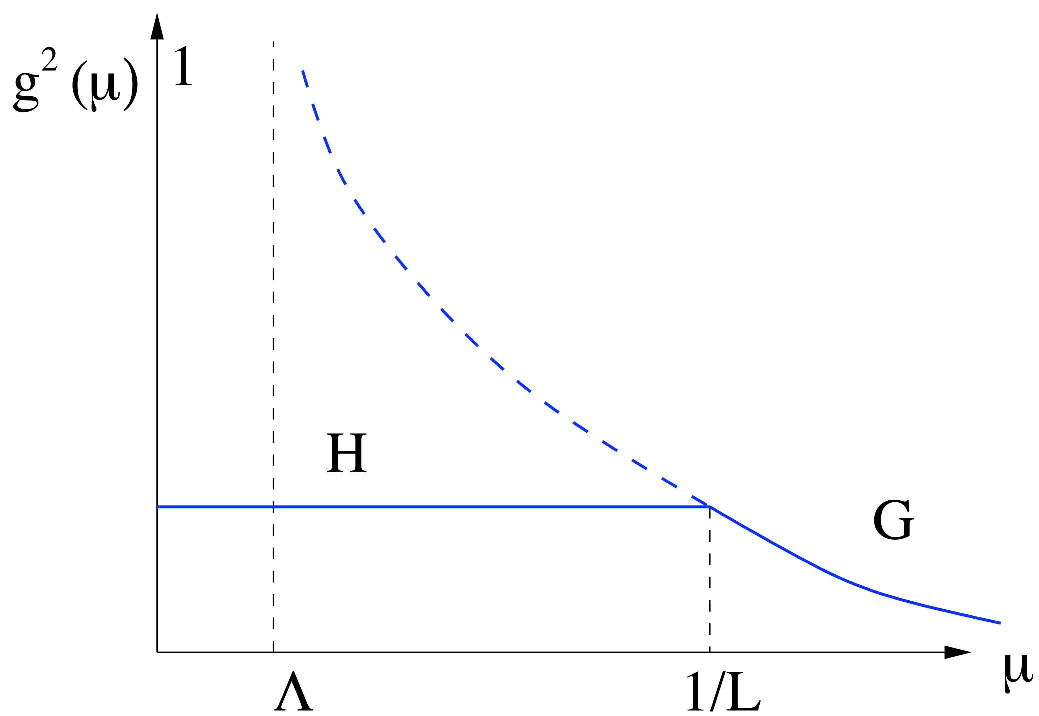
At weak coupling, the fluctuations are small, a "Higgs regime"

$$SU(N) \to [U(1)]^{N-1}$$

Georgi-Glashow model with compact adjoint Higgs field.

Compactness implies N types of monopoles, rather than N-1.

Perturbation theory



$$G = SU(2), \quad H = U(1)$$

IR in perturbation theory is a free theory of "photons". Is this perturbative fixed point destabilized non-perturbatively?

Reminder: Abelian duality and Polyakov model

Free Maxwell theory is dual to the free scalar theory.

$$F = *d\sigma$$

The masslessness of the dual scalar is protected by a **continuous shift symmetry**

$$U(1)_{\text{flux}}: \ \sigma \to \sigma - \beta$$

Noether current of dual theory:

Topological current vanishes by Bianchi identity.

$$\mathcal{J}_\mu = \partial_\mu \sigma = \tfrac{1}{2}\epsilon_{\mu\nu\rho}F_{\nu\rho} = F_\mu$$

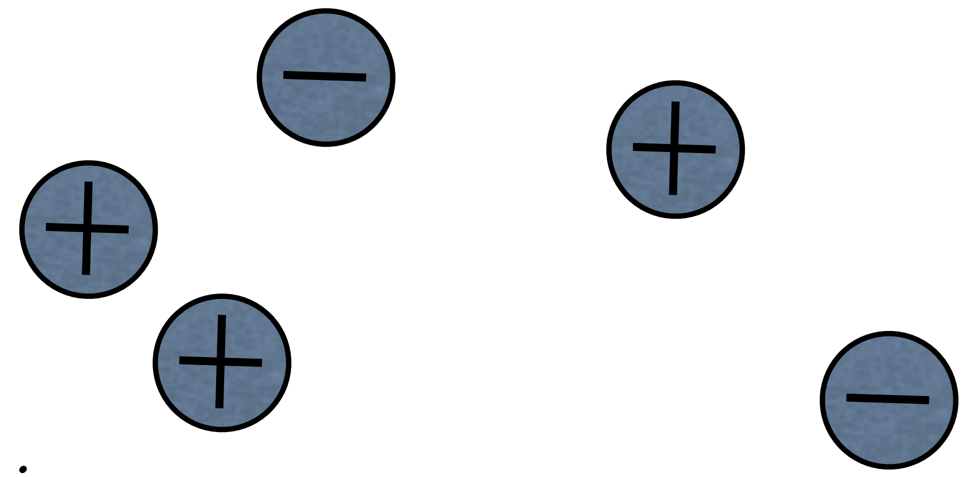
Its conservation implies the absence of magnetic monopoles in original theory

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = 0$$

Proliferation of monopoles

The presence of the monopoles in the original theory implies reduction of the continuous shift symmetry into a discrete one. Polyakov mechanism.

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = \rho_m(x)$$



The dual theory

$$L = \frac{1}{2}(\partial\sigma)^2 - e^{-S_0}(e^{i\sigma} + e^{-i\sigma})$$

Physics of Debye mechanism

Discrete shift symmetry: $\sigma \to \sigma + 2\pi$

$U(1)_{\mathrm{flux}}$ if present, forbids (magnetic) flux carrying operators.

Topological excitations in QCD(adj)

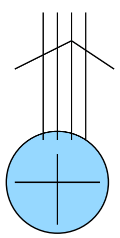
$$\left(\int_{S^2} F,\ \int_{R^3\times S^1} F\tilde{F}\right)$$

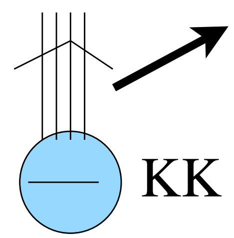
Magnetic Monopoles

Magnetic Bions

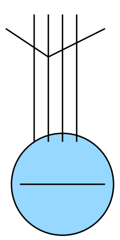
relevant index theorems
Callias 78
Nye-A.M.Singer, 00
E. Weinberg 80
Poppitz, MU 08
Atiyah-M.I.Singer 75

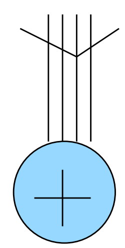
fermionic zero modes


BPS
(1, 1/2)


KK
(–1, 1/2)

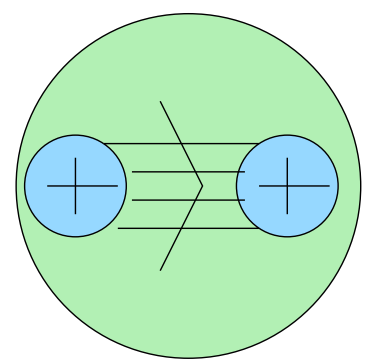
$$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$$

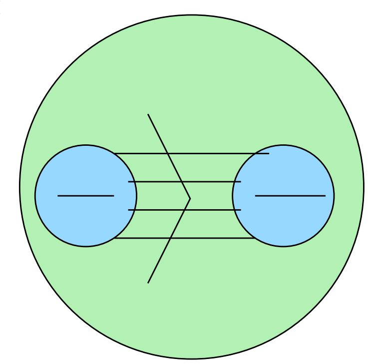

$\overline{\text{BPS}}$
(–1, –1/2)


$\overline{\text{KK}}$
(1, –1/2)

$$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$$

$$(\mathbb{Z}_2)_*$$


(2,0)


(–2, 0)

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

Discrete shift symmetry : $\quad \sigma \longrightarrow \sigma + \pi \qquad \psi^I \longrightarrow e^{i\frac{2\pi}{8}} \psi^I$

Crucial earlier work: van Baal et.al. and Lu, Yi, 97

Dual Formulation of QCD(adj)

$$L^{\text{dQCD}} = \frac{1}{2}(\partial\sigma)^2 - b\, e^{-2S_0} \cos 2\sigma + i\bar{\psi}^I \gamma_\mu \partial_\mu \psi_I + c\, e^{-S_0} \cos\sigma (\det_{I,J} \psi^I \psi^J + \text{c.c.})$$

magnetic bions

Non-selfdual

magnetic monopoles

Self-dual

Same mechanism in N=1 SYM.

Also see Hollowood, Khoze, ... 99

Earlier in N=1 SYM, the bosonic potential was derived using supersymmetry and SW-curves, F, M theories, field theory methods. However, the physical origin of it remained elusive till this work.

$$m_\sigma \sim \frac{1}{L} e^{-S_0(L)} = \frac{1}{L} e^{-\frac{8\pi^2}{g^2(L)N}} = \Lambda(\Lambda L)^{(8-2N_f^W)/3} \,,$$

Proliferation of magnetic bions

Increasing for Nf<4

Decreasing for 4<Nf<5.5

To the surprise of the past

Theory	Confinement mechanism on $R^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-Singer, E. Poppitz, MU	Index for instanton $\mathcal{I}_{\text{inst.}}$ Atiyah-Singer	(Mass Gap)^2
YM	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F)	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM/QCD(Adj)	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF)	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS)	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S)	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
$SU(2)$ YM $I = \frac{3}{2}$	magnetic quintets	$[4, 6]$	10	e^{-5S_0}
chiral $[SU(N)]^K$	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\text{F}}$	bions and a monopole	$[1, 1, , \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\text{AS} + (N-4)\overline{\text{F}}$	e^{-2S_0}, e^{-S_0},
$S + (N+4)\overline{\text{F}}$	bions and triplets	$[1, 1, , \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\text{AS} + (N+4)\overline{\text{F}}$	e^{-2S_0}, e^{-3S_0},

More refined data

Table 1: Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$.

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Theory	Confinement mechanism on $R^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-Singer, E. Poppitz, MU	Index for instanton $\mathcal{I}_{\text{inst.}}$ Atiyah-Singer	$(\text{Mass Gap})^2$
YM LGY, MU, 08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) MS, MU, 08	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM/QCD(Adj) MU, 07	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF) MS, MU, 08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS) MS, MU, 08	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) EP, MU, 09	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
$SU(2)$ YM $I = \frac{3}{2}$ EP, MU, 09	magnetic quintets	$[4, 6]$	10	e^{-5S_0}
chiral $[SU(N)]^K$ MS, MU, 08	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\text{F}}$ MS, MU, 08	bions and a monopole	$[1, 1, , \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\text{AS} + (N-4)\overline{\text{F}}$	e^{-2S_0}, e^{-S_0},
$S + (N+4)\overline{\text{F}}$ EP, MU, 09	bions and triplets	$[1, 1, , \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\text{AS} + (N+4)\overline{\text{F}}$	e^{-2S_0}, e^{-3S_0},

More refined data

Table 1: Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$.

Conformality or confinement:

Conceptually, two problem of out-standing importance in gauge theories:

Mechanisms of confinement and conformality:
What distinguishes two theories, one just below the conformal boundary and confines, and the other slightly above the conformal window boundary? In other words, why does a confining gauge theory confine and why does an IR-CFT, with an almost identical microscopic matter content, flows t a CFT?

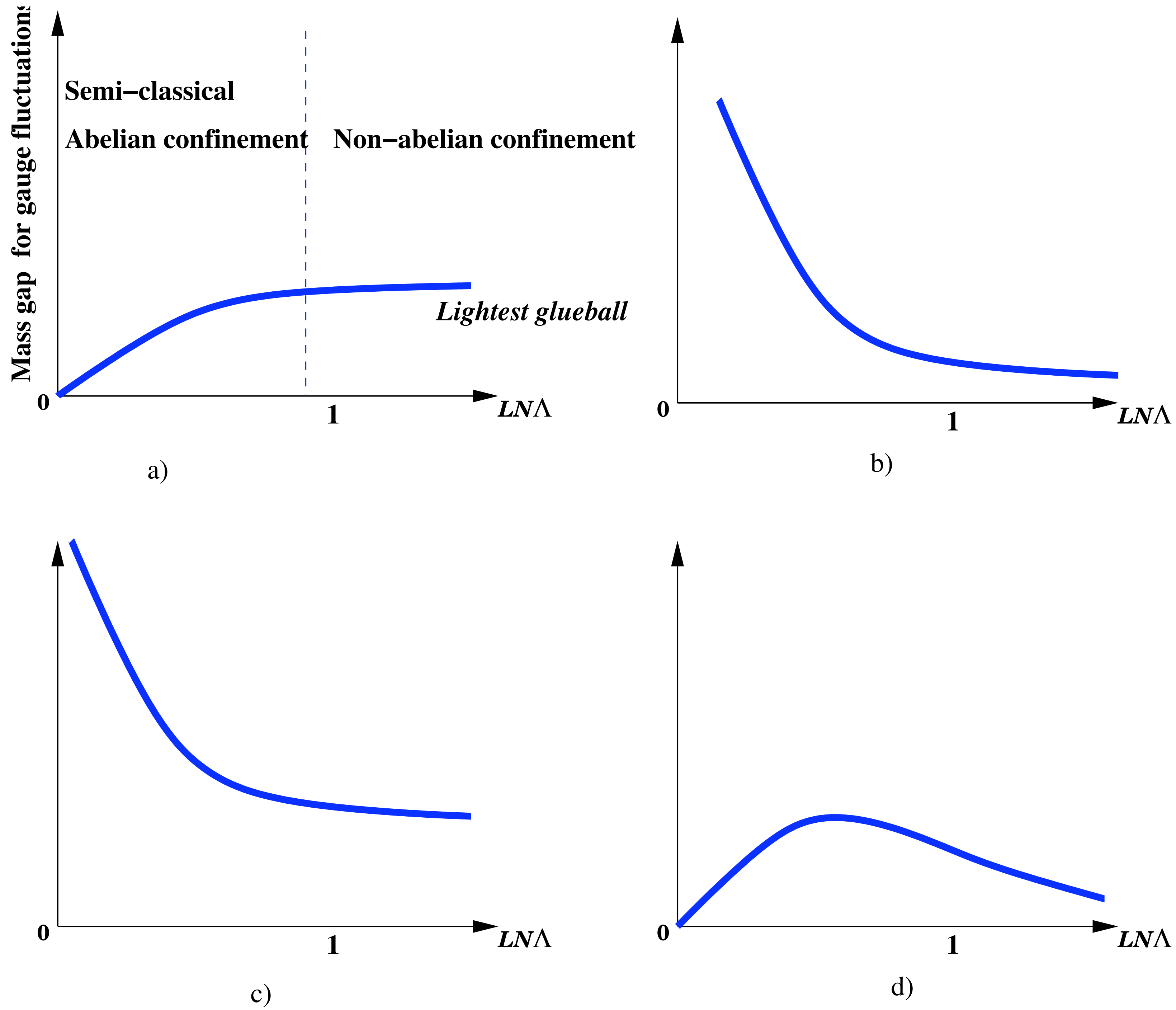
Lower boundary of conformal window: What is the physics determining the boundary of conformal window?

Map the problem to the mass gap for gauge fluctuations:

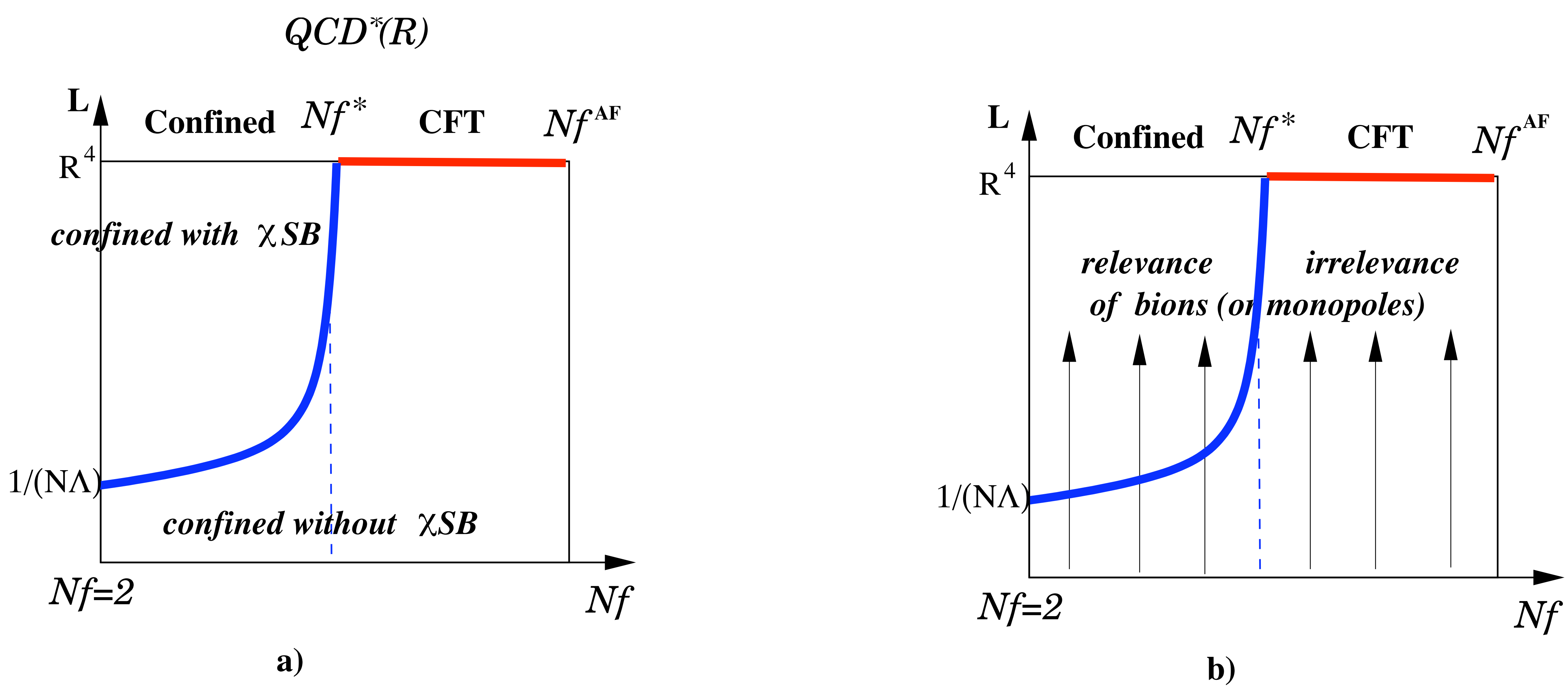
$$m^{-1}_{\text{gauge fluc.}}(\mathbf{R}^4) = \begin{cases} \text{finite} & N_f < N_f^* \qquad\qquad \text{confined} \\ \infty & N_f^* < N_f < N_f^{AF} \qquad \text{IR}-\text{CFT} \end{cases}$$

A priori, not a smart strategy.

Mass gap for gauge fluctuations



Main Idea of our proposal

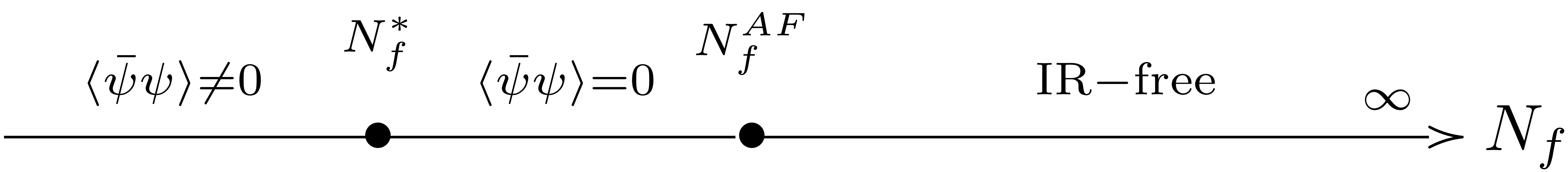


Crucial data: Index theorem on R3*S1, the knowledge of mechanism of confinement, and one-loop beta function.

QCD(F/S/AS/Adj):Estimates and comparisons

Below, I will present the estimates based on this idea and compare it various other approaches. In particular:

1) Truncated SD (ladder, rainbow) approximation. M.E.Peskin, Chiral Symmetry And Chiral Symmetry Breaking, Les Houches, 1982 (Up-to date review)



QCD(F): Appelquist, Lane, Mahanta, and Miransky
Two-index cases: Sannino, Dietrich.

2) NSVZ-inspired conjecture: Sannino, Ryttov.

Crucial data for 1) and 2): Two-loop (or conjectured all orders) beta function, anomalous dimension of fermion bilinear.

3)World-line formalism: Armoni.

Caveat: chiral gauge theories.

QCD(S/AS/Adj):Estimates and comparisons

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
∞	4	4.15	2.75/3.66	5.5

Table 1: Estimates for lower boundary of conformal window in QCD(S), $N_f^* < N_f^D < 5.5\left(1 - \frac{2}{N+2}\right)$.

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
4	8	8.10	5.50/7.33	11
5	6.66	6.80	4.58/6.00	9.16
6	6	6.15	4.12/5.5	8.25
10	5	5.15	3.43/4.58	6.87
∞	4	4.15	2.75/3.66	5.50

Table 2: Estimates for lower boundary of conformal window in QCD(AS), $N_f^* < N_f^D < 5.5\left(1 + \frac{2}{N-2}\right)$.

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\ \gamma = 1$	N_f^{AF}
any N	4	4.15	2.75/3.66	5.5

Table 3: Estimates for lower boundary of conformal window in QCD(adj), $N_f^* <$

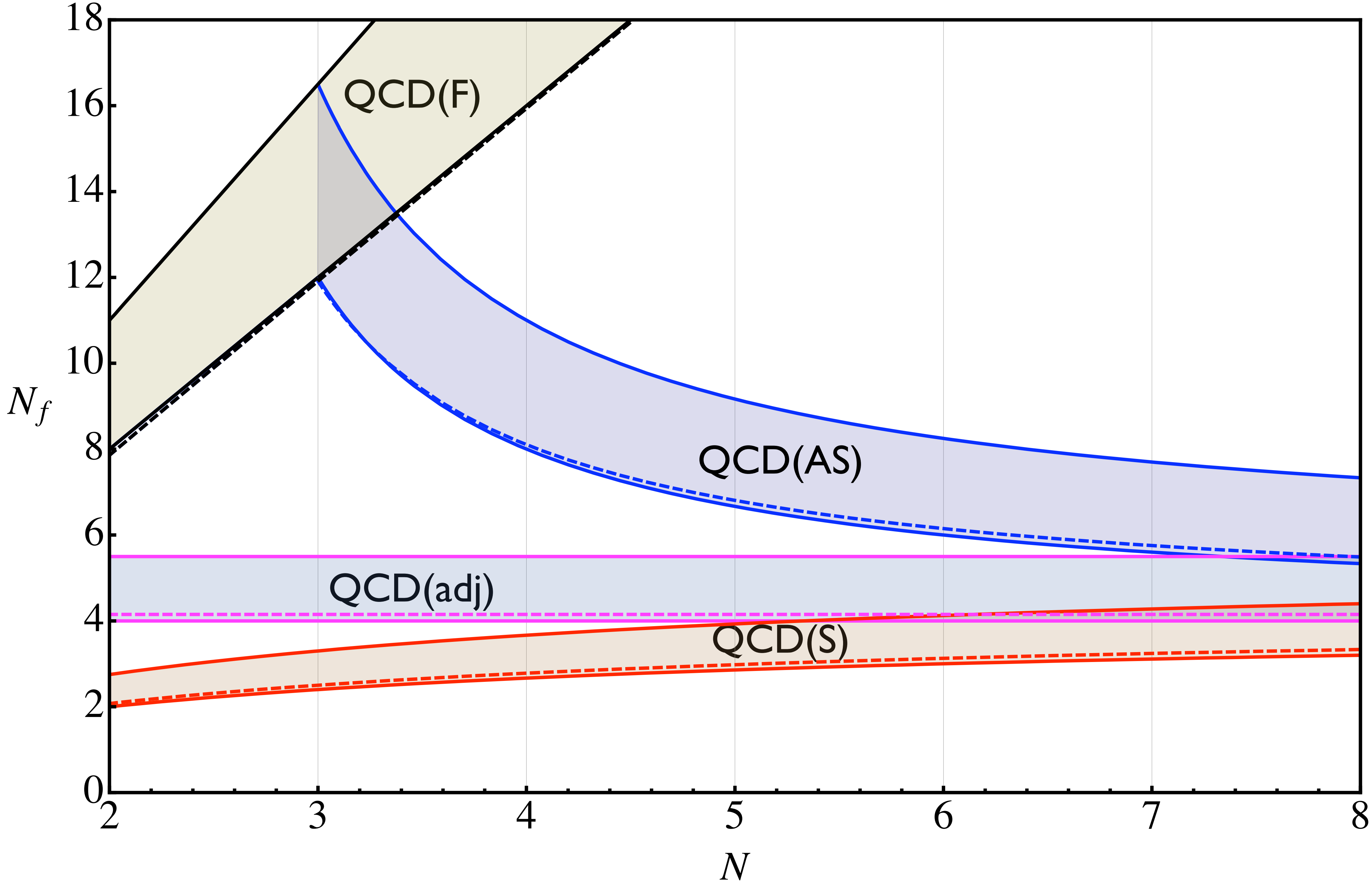
QCD(F)

N	D.T. 1a/1c	Ladder (SD)-approx.	Functional RG	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
2	5/8	7.85	8.25	5.5/7.33	11
3	7.5/12	11.91	10	8.25/11	16.5
4	10/16	15.93	13.5	11/14.66	22
5	12.5/20	19.95	16.25	13.75/18.33	27.5
10	25/40	39.97	n/a	27.5/36.66	55
∞	$2.5N/4N$	$4N$	$\sim (2.75 - 3.25)N$	$2.75N/3.66N$	$5.5N$

Table 1: Estimates for lower boundary of conformal window for QCD(F), $N_f^* < N_f^D < 5.5N$

The above estimates from DT are for class a and a+c, respectively.

Estimates of the conformal window from deformation theory



Dashed line: Truncated SD (ladder, rainbow) approximation.

QCD(F): Appelquist, Lane, Mahanta, Miransky.....
Two-index: Sannino et.al.

BUT WHY?

How can we relate perturbation theory to non-pert. physics?

$$\gamma(g^2) = \frac{3}{2}\frac{(g^2 N)}{8\pi^2}[1 + O(g^2 N)].$$ Anomalous dimension of fermion bilinear

$$\text{If } \gamma(L) < 1, \text{ no } \chi\text{SB}$$

Monopole action: $S_0(L) = \dfrac{8\pi^2}{g^2(L)N}$

$$\begin{array}{ll} \gamma(L) \ll 1 \Rightarrow S_0(L) \gg 1 \Rightarrow e^{-S_0} \ll 1, & \text{dilute gas of monopoles and bions} \\ \gamma(L) \sim 1 \Rightarrow S_0(L) \sim 1 \Rightarrow e^{-S_0} \sim 1, & \text{non} - \text{dilute} \end{array}$$

means non-abelian confinement cannot set-in.

$$\gamma(\overline{\psi}\psi)S_0 = \gamma(g^2(L))S_0(g^2(L)) \sim 1$$

Needs refinement, but it seems to be on the right path.

Conclusions

- There is now a window through which we can look into non-abelian gauge theories and understand their internal goings-on. Whether the theory is chiral, pure glue, or supersymmetric is immaterial. We always gain a semi-classical window (in some theories smoothly connected to R4 physics.)

- Deformation theory is complementary to lattice gauge theory. Sometimes lattice is more powerful, and sometime otherwise. Currently, DT is the only dynamical framework for chiral gauge theories. It may also be more useful in the strict chiral limit of vector-like theories.

- Most important: We learned the existence of a large class of new non-self dual topological excitations through this program in the last two years.

- Shed light into the mechanisms of conformality and confinement in gauge theories. This is tied with the (IR)relevance of topological excitations.